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QUICK REVISION

FORMULA SHEET

for

GATE-AE FLUID MECHANIC

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FLUID MECHANICS

FLUID BASICS

Note:

1. For a static fluid, shear force is zero.
2. Fluids only show resistance to compressive loads.

$$k = \text{Bulk Modulus} = \frac{\text{Hydro static stress}}{\text{Volumetric strain}} = \frac{-dp}{dV/V}$$

$$\beta = \frac{1}{k}$$

$(\beta) = \text{Compressibility}$

For Incompressible fluid $\beta = 0$

Reason of viscosity(μ) in liquids → Cohesive force (Molecular Bonding)

Reason of viscosity(μ) in gases → Molecular Collision.

- Temperature $\uparrow \Rightarrow \mu_{liquid} \downarrow \Rightarrow \nu_{liquid} \downarrow$

  Where $\nu_{liquid} \Rightarrow$ Kinematic viscosity of liquid.
- Temperature $\uparrow \Rightarrow \mu_{gas} \uparrow \Rightarrow \nu_{gas} \uparrow\uparrow$
- Pressure $\uparrow \Rightarrow \mu_{liquid}$: same $\Rightarrow \nu_{liquid}$: same
- Pressure $\uparrow \Rightarrow \mu_{gas}$: same $\Rightarrow \nu_{liquid}$: $\downarrow$

Newton's Law of Viscosity:

$\tau = \mu \dfrac{du}{dy}$ $\quad \tau$ = shear stress

where μ = Viscosity/Dynamic viscosity

Unit of μ → kg/(meter-sec)or pascal-sec

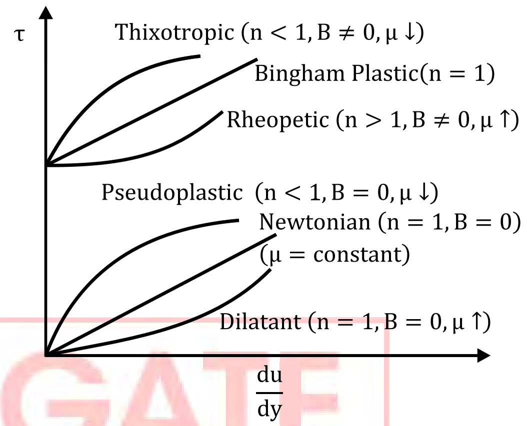
$$\frac{1\text{kg}}{\text{meter-sec}} = 10 \text{ poise}$$

1poise = 0.1pascal-sec

Kinematic viscosity $(\nu) = \text{meter}^2/\text{sec}$

$$\frac{1\text{m}^2}{\text{sec}} = 10^4 \text{stokes}$$

Stokes Cgs unit of kinematic viscosity



For Non-Newtonian Fluid:

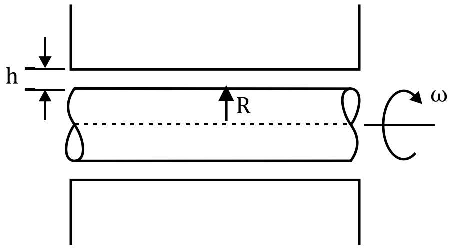
$$\tau = A\left(\frac{du}{dy}\right)^n + B$$

Ideal Fluid:

- Non viscous
- Incompressible

Power Lost in Bearing due to Fluid Friction:

Case 1:



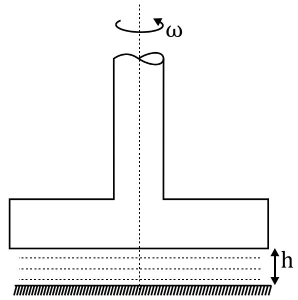
$$\text{Power} = \frac{2\pi\mu w^2 L R^3}{h}$$

R = Radius of shaft

h = oil film gap

w = angular velocity of shaft

Case 2:



$$\text{Torque} = \frac{\pi \mu w R^4}{2h}$$

FLUID STATICS

$p_{absolute} = p_{atm} + p_{gauge}$

Hydrostatic Law:

$$\frac{dp}{dh} = -\rho g$$

Hydrostatic Forces:

$F = \rho g \bar{h} A$

$\bar{h}$ = Distance of centre of gravity from free liquid surface.

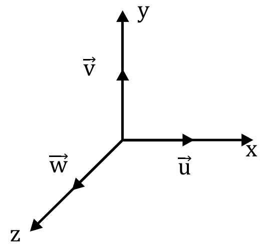
Location of hydrostatic forces

$$\bar{h}_{cp} = \bar{h} + \frac{I_{Gx} \sin^2\theta}{A\bar{h}}$$

$\bar{h}_{cp}$ = Distance of pressure force from free liquid surface.

FLUID KINEMATICS

Acceleration:



$$a = a_x\hat{\imath} + a_y\hat{\jmath} + a_z\hat{k}$$

$$a_x = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} + \frac{\partial u}{\partial t}$$

$$a_y = u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} + \frac{\partial v}{\partial t}$$

$$a_z = u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} + \frac{\partial w}{\partial t}$$

$$\frac{\partial u}{\partial t}, \frac{\partial v}{\partial t}, \frac{\partial w}{\partial t}$$

These are local component or temporal component of acceleration.

$$\vec{a} = \vec{a}_{convective} + \vec{a}_{temporal}$$

Angular Velocity:

$$\vec{V} = u\hat{\imath} + v\hat{\jmath} + w\hat{k}$$

$$w = \frac{1}{2}(\nabla \times \vec{v})$$

$$w_x = \frac{1}{2}\left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right)$$

$$w_y = \frac{1}{2}\left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right)$$

$$w_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$$

Vorticity $(\boldsymbol{\Omega}) = \boldsymbol{\nabla} \times \vec{\mathbf{v}} = \mathbf{2}\vec{\mathbf{w}}$

Linear strain rate:

$$\varepsilon_{\dot{x}x} = \frac{\partial u}{\partial x}, \varepsilon_{\dot{y}y} = \frac{\partial v}{\partial y}, \varepsilon_{\dot{z}z} = \frac{\partial w}{\partial z}$$

$$\varepsilon_v = \varepsilon_{\dot{x}x} + \varepsilon_{\dot{y}y} + \varepsilon_{\dot{z}z}$$

(Volumetric strain rate) or Dilation

$\varepsilon_{\dot{v}} = 0$ For incompressible flow

Shear Strain Rate:

$$\varepsilon_{xy} = \frac{1}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)$$

$$\varepsilon_{xz} = \frac{1}{2}\left(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}\right)$$

$$\varepsilon_{yz} = \frac{1}{2}\left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right)$$

$u =$ x compoent of velocity

$v =$ y component of velocity

$w =$ z componet of velocity

Shear deformation rate /Angular deformation rate:

$$\gamma_{xy} = \left[\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right]$$

Circular (Γ):

$$\Gamma = \int \vec{V}\cdot\vec{dr} \text{ (or) } \Omega \times \text{Area (or)} \iint \Omega dA$$

$\Omega =$ Vorticity

Velocity Potential Function (ϕ) for 3D flow.

$$u = \frac{\partial\phi}{\partial x}$$

$$v = \frac{\partial\phi}{\partial y}$$

$$w = \frac{\partial\phi}{\partial z}$$

In Polar coordinator:

$$u_r = \frac{\partial\phi}{\partial r}$$

$$V_o = +\frac{1}{r}\frac{\partial\phi}{\partial\theta}$$

- If ϕ exists $\Rightarrow$ Flow is irrotational
- If ϕ satisfies Laplace equation $\Rightarrow$ Flow is possible.
- Slope of equipotential line

$$\Rightarrow -\frac{u}{v} \Rightarrow \frac{dy}{dx}$$

Stream Function (ψ) (For 2D Flow):

$$u = \frac{\partial\psi}{\partial y}$$

$$v = -\frac{\partial\psi}{\partial x}$$

$$u_r = \frac{1}{r}\frac{\partial\psi}{\partial\theta}$$

$$V_\theta = -\frac{\partial\psi}{\partial r}$$

- If ψ exists $\Rightarrow$ Flow is possible
- If ψ satisfied Laplace equation $\Rightarrow$ Flow is irrotational
- Slope of streamline

$$\Rightarrow \frac{dy}{dx} = \frac{v}{u}$$

Laplace Equation:

$$\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} + \frac{\partial^2\phi}{\partial z^2} = 0$$

$$\nabla^2\phi = 0$$

∇^2 (any variable) $= 0$ (Laplace Equation)

Discharge Per unit length $= |\psi_1 - \psi_2|$

FLUID DYNAMICS

$F_{pressure}, F_{gravity}, F_{viscous}$

$\Rightarrow$ Use Navier-strokes equation

$F_{pressure}, F_{gravity} \Rightarrow$ Euler equation

$F_{pressure}, F_{gravity}$ and incompressible

$\Rightarrow$ Bernoulli's equation

Ideal Bernoulli's Equation: (Can be used for rotational flow also but Flow must be along same streamline)

$$\frac{p}{\rho g} + \frac{V^2}{2g} + z = \text{Constant}$$

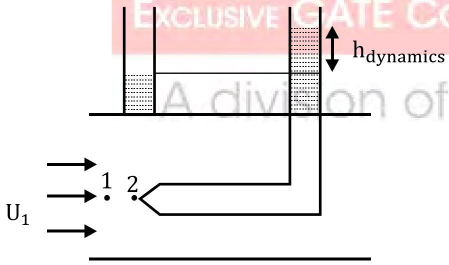
Assumptions:

- Non viscous
- Steady flow
- Incompressible flow of incompressible fluid
- Irrotational flow.

For Practical use (Bernoulli's equation with losses)

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + Z_2 + h_{losses}$$

Pitot Tube: (Measures stagnation pressure head).



$$u_1 = \sqrt{2g\, h_{dynamic}}$$

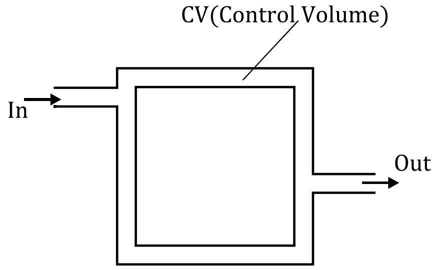
$$h_{dynamic} = x\left(\frac{\rho_m}{\rho} - 1\right)$$

x = Manometric deflection

(or)

$$h = \left(\frac{p_2}{\rho g} + Z_2\right) - \left(\frac{p_1}{\rho g} + Z_1\right)$$

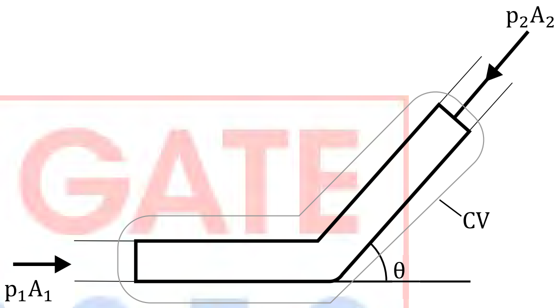
Impulse Momentum Equation:



$$\vec{F}_{net} + \sum \dot{m}_i v_i = \left(\frac{d(mv)}{dt}\right)_{cv} + \sum \dot{m}_o \vec{v}_o$$

$f_{net} = \dot{m}_o V_o - \dot{m}_i v_i$ (for steady flow)

If pressure forces considered, then



$$F_x + p_1A_1 - p_2A_2\cos\theta = \dot{m}[V_2\cos\theta - V_1]$$

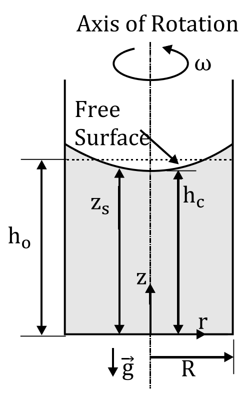
$$F_y - p_2A_2\sin\theta = \dot{m}[V_2\sin\theta]$$

f_{net} or $F_x, F_y \rightarrow$ Force exerted by pipe bend or structure on fluid element.

Rotational flow in a Cylindrical Container:

Equation of motion for vortex flow

$$dP = \rho r\omega^2 dr - \rho g\, dz$$

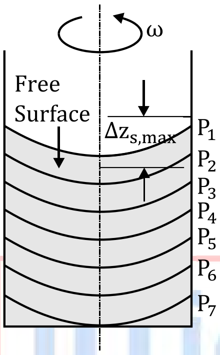


the equation of the free surface

$$z_s = h_0 - \frac{\omega^2}{4g}(R^2 - 2r^2)$$

The maximum vertical height occurs at the edge where r = R, and the maximum height difference between the edge and the center of the free surface is

$$\Delta z_{s,max} = z_s(R) - z_s(0) = \frac{\omega^2}{2g}R^2$$



where h_0 is the original height of the fluid in the container with no rotation

FLUID THROUGH PIPES

Darcy Weisbeck Equation:

$$h_f = \frac{FlV^2}{2gd}$$ Valid for Laminar and Turbulent

F = Friction factor $= \frac{64}{Re}$ Laminar flow

Re = Reynolds Number $= \frac{\rho V D_H}{\mu}$

P_H = Hydraulic daimeter

Laminar Flow:

1. **Through Pipe:**

(A) $\tau = -\frac{r}{2}\left(\frac{\partial p}{\partial x}\right)$

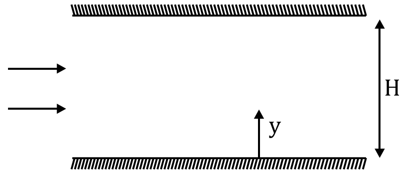
(B) $u = -\frac{1}{4\mu}\left(\frac{\partial p}{\partial x}\right)[R^2 - r^2]$

(C) $u_{max} = \frac{1}{4\mu}\left(-\frac{\partial p}{\partial x}\right)(R^2)$

(D) $V_{avg} = \frac{V_{max}}{2}$

2. **Laminar Flow Through Two Parallel Plate:**

Case A:



$$\tau = -\frac{\partial p}{\partial x}\left(\frac{H}{2} - y\right)$$

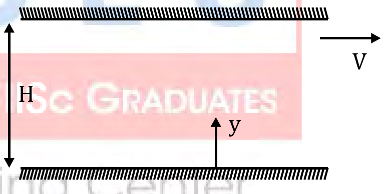
$$\tau_{wall} = -\frac{\partial p}{\partial x}\left(\frac{H}{2}\right)$$

$$u = \frac{1}{2\mu}\left(\frac{\partial p}{\partial x}\right)(Hy - y^2)$$

$$u_{max} = -\frac{1}{8\mu}\left(\frac{\partial p}{\partial x}\right)(H^2)$$

$$u_{avg} = \frac{2V_{max}}{3}$$

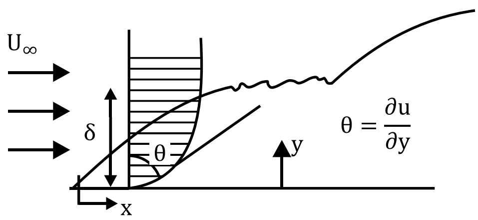
Case B: (couette flow)



$$u = -\frac{1}{2\mu}\left(\frac{\partial p}{\partial x}\right)[Hy - y^2] + \frac{Vy}{H}$$

BOUNDARY LAYER THEORY:

Boundary layer region is highly viscous and rotational region of flow⇒ so Bernoulli's equation not applicable.



Boundary Condition:

at $x = 0, \delta = 0$

at $y = 0, u = 0$

at $y = \delta, \dfrac{\partial u}{\partial y} = 0$

at $y = 0, \dfrac{\partial^2 u}{\partial y^2} = 0$

$\Rightarrow \delta^*$[Displacement thickness]

$$= \int_0^\delta \left(1 - \frac{U}{U_\infty}\right) dy$$

Loss pf mass flow rate $= (\rho\delta^* U_\infty . b)$

$b =$ Width of plate

Momentum Thickness $[\theta]$

$$\theta = \int_0^\delta \frac{U}{U_\infty}\left(1 - \frac{U}{U_\infty}\right) dy$$

$F_{drag} = \rho A U_\infty^2 \Rightarrow \rho(\theta \times b) \times U_\infty^2$

Energy Thickness $[\delta^{**}]$

$$= \int_0^\infty \frac{U}{U_\infty}\left(1 - \left(\frac{U}{U_\infty}\right)^2\right) dy$$

Shape Factor $= \dfrac{\delta^*}{\theta}$

$\delta > \delta^* > \delta^{**} > \theta$ valid for all velocity profile.

Von Karman Momentum Integral Equation.

$\dfrac{\tau_o}{\rho U_\infty^2} = \dfrac{d\theta}{dx}$ Valid for laminar and turbulent

Used when velocity profile given in the problem.

$T_o = \mu \left(\dfrac{\partial u}{\partial y}\right)_{y=0}$

Blasius Equation: (Used when velocity profile is not given)

Laminar Flow:

$(Re < 5 \times 10^5)$

$$\delta = \frac{5x}{\sqrt{Re_x}}$$

$$C_D = \frac{1.328}{\sqrt{Re_L}}$$

$$CF_x = \frac{0.664}{\sqrt{Re_x}}$$

$Re =$ Reynold's Number

$C_D =$ Average drag coefficient.

$CF_x =$ Local Drag Coefficient.

$$\delta^* = \frac{1.72(x)}{\sqrt{Re_x}}$$

$$\theta = \frac{0.664(x)}{\sqrt{Re_x}}$$

Turbulent Flow: (Used when 1/7th law given in problem) $\dfrac{U}{U_\infty} = \left(\dfrac{y}{\delta}\right)^{1/7}$

$$\delta = \frac{0.16x}{(Re)^{\frac{1}{7}}}$$

$$\delta^* = \frac{0.02(x)}{(Re)^{\frac{1}{7}}}$$

$$\theta = \frac{0.016(x)}{(Re)^{\frac{1}{7}}}$$

Boundary Layer Separation:

$$\left.\frac{\partial u}{\partial y}\right|_{y=0} = 0 \text{ and } \frac{\partial p}{\partial x} > 0$$

DIMENSIONAL ANALYSIS

Dynamic Similarity:

$$\frac{F_{inertia})_m}{F_{inertia})_p} = \frac{F_{viscous})_m}{F_{viscous})_p}$$

$$\left.\frac{F_i}{F_v}\right)_m = \left.\frac{F_i}{F_v}\right)_p$$

$$Re)_m = Re)_p$$

Reynold's number for model and prototype shall be same.

$F_{inertia} = \rho L^2 V^2$

$F_{viscous} = \mu VL,$

where μ = Viscosity

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